Multiple Phase Flow

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Introduction

The **drag force** is a resistive force exerted by a fluid (gas or liquid) on a particle, bubble, or droplet moving relative to the fluid. In multiphase flow systems, understanding and quantifying the drag force is critical for predicting phase interactions, particle dynamics, and flow behavior. **Drag force correlations** provide empirical or theoretical relationships to calculate the drag force under various flow conditions.

Key Concepts

1. Drag Force (F_D)

The drag force acting on a particle is given by:

$$F_D = \frac{1}{2} C_D \rho_f A\, u_r^2$$

Where:

- C_D: Drag coefficient (dimensionless)
- ρ_f: Fluid density (kg/m3\text{kg/m}^3)
- A: Projected area of the particle (m2\text{m}^2)
- u_r: Relative velocity between the particle and fluid (m/s\text{m/s})

2. Drag Coefficient (C_D)

The drag coefficient depends on:

- **Reynolds number (Re)**: Ratio of inertial to viscous forces.

$$Re = \frac{\rho_f u_r D}{\mu}$$

Where:

- D: Particle diameter (m)
- μ: Fluid viscosity (Pa.s)

- **Flow Regime**: Laminar, transitional, or turbulent flow around the particle.
- **Shape and Orientation**: Affects the effective drag area.

Drag Force Correlations

1. Stokes' Law (Low Reynolds Number: Re < 1)

For small, spherical particles in laminar flow:

$$F_D = 3\pi\mu D u_r$$

$$C_D = \frac{24}{Re}$$

2. Intermediate Reynolds Number (1 < Re < 1000)

For transitional flow:

$$C_D = \frac{24}{Re}\left(1 + 0.15Re^{0.687}\right)$$

This correlation accounts for non-linear drag effects as Re increases.

3. High Reynolds Number (Re > 1000)

For turbulent flow around the particle:

$$C_D = 0.44$$

This value is nearly constant due to turbulent wake formation.

Empirical Correlations for Drag Coefficient

Schiller-Naumann Correlation: Valid for Re ≤ 1000:

$$C_D = \frac{24}{Re}(1 + 0.15Re\text{^}0.687)$$

Haider and Levenspiel Correlation: Applicable for a wider range of Re:

$$C_D = \frac{24}{Re}(1 + 0.15Re^{0.687}) + \frac{0.42}{1 + 42500Re^{-1.16}}$$

Gidaspow Correlation: Frequently used in gas-solid flows (e.g., fluidized beds):

$$C_D = \frac{24}{Re}(1 + 0.15Re^{0.687}) \quad \text{for Re} < 1000$$

$$C_D = 0.44 \quad \text{for Re} \geq 1000$$

Drag Force in Multiphase Systems

1. Spherical Particles

For spherical particles, the correlations above apply directly, as they assume isotropic drag characteristics.

2. Non-Spherical Particles

For non-spherical particles, shape factors (ϕs\phi_s) are introduced:

$$C_D = C_{D,spherical} \cdot \phi s$$

Where $\phi s > 1$ accounts for increased drag due to non-spherical shapes.

3. Bubble or Droplet Dynamics

For bubbles or droplets, the drag force depends on deformation and wake dynamics:

- Small bubbles: Use Stokes' law.
- Large or deformable bubbles: Drag increases with wake turbulence.

Factors Affecting Drag Force

Reynolds Number (Re):

- Determines flow regime around the particle.
- Directly influences C_D.

Particle Size and Shape:

- Larger or irregular particles experience higher drag.

Fluid Properties:

- Density (ρ_f) and viscosity (μ) influence the relative importance of inertial and viscous forces.

Relative Velocity (u_r):

- Higher relative velocities increase the drag force exponentially.

Flow Regime:

- Laminar, transitional, or turbulent flow alters the drag coefficient.

Applications

Fluidized Beds:

- Drag force is critical in maintaining particle suspension and bed expansion.

Pneumatic Conveying:

- Predicts particle transport in gas-solid systems.

Sedimentation:

- Drag force determines settling velocity of particles in liquids.

Aerosol and Spray Dynamics:

- Drag influences the trajectory and evaporation of droplets.

Multiphase Reactors:

- Key to modeling gas-liquid-solid interactions.

Example Calculation

Problem:

A spherical particle (D = 0.005 m) moves through air (ρ_f=1.2 kg/m^3, μ=1.8×10^{-5} Pa.s at a relative velocity of u_r=10 m. Calculate the drag force using the Schiller-Naumann correlation.

Solution:

Reynolds Number:

$$Re = \frac{\rho_f u_r D}{\mu}$$

$$Re = 1.2 \cdot 10 \cdot 0.0051.8 \times 10^{-5} = 3333.33$$

Drag Coefficient (C_D): Since Re > 1000, the drag coefficient is constant:

$$C_D = 0.44$$

Drag Force:

$$F_D = \frac{1}{2} C_D \rho_f A u_r^2$$

The projected area:

$$A = \frac{\pi D^2}{4} = \frac{\pi(0.005)^2}{4} = 1.963 \times 10^{-5}\ \mathrm{m}^2$$

Substituting:

$$F_D = \frac{1}{2} \cdot 0.44 \cdot 1.2 \cdot 1.963 \times 10^{-5} \cdot 10^2$$

$$F_D = 0.0518\ \mathrm{N}$$

Conclusion

The **drag force correlation** is a fundamental tool for analyzing particle-fluid interactions in multiphase systems. By using empirical and theoretical correlations, engineers can predict drag forces accurately for a wide range of applications, from sedimentation to fluidized beds and beyond.